

Commuting Matrices

Part I

We have seen that, in general, multiplication by matrices is not commutative. For Part I of your assignment, choose two 2x2 matrices that demonstrate that multiplication of matrices is not generally commutative. Then, explain in your own words what it is about matrix multiplication that makes it non-commutative. Other than the obvious fact that the answers are different, you should explain in a brief paragraph WHY they are different.

(10 points)

Part II

(20 points)

Although matrix multiplication is not commutative in general, there may be a particular class of matrices with which a matrix will commute multiplicatively.

For example:

To find out the class of matrices that will commute with $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$, we can set

up two situations:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2a-b & -3a+b \\ 2c-d & -3c+d \end{bmatrix}$$

and

$$\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a-3c & 2b-3d \\ -a+c & -b+d \end{bmatrix}$$

Therefore, for $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$ to commute with a matrix, the following must be

true...

- 1) $2a - b = 2a - 3c$
- 2) $-3a + b = 2b - 3d$
- 3) $2c - d = -a + c$
- 4) $-3c + d = -b + d$

Working with equation #1, it is clear that $b = 3c$. Equation #4 leads to the same conclusion. Equation #3 shows that $c = d - a$ or, equivalently, $d = a + c$. The

idea is to express the class of matrices that commute with $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$ using the

fewest possible variables. So, we can describe this class of matrices that

commute with $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$ as:

$$\begin{bmatrix} a & b \\ \frac{1}{3}b & a + \frac{1}{3}b \end{bmatrix} \text{ or } \begin{bmatrix} a & 3c \\ c & a + c \end{bmatrix} \text{ or } \begin{bmatrix} a & 3d - 3a \\ d - a & d \end{bmatrix}$$

These all describe the same relationship between the entries of the matrix and each one is a correct solution to the problem of describing the class of matrices

that commutes with $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$.

Your assignment in Part II is to describe the class of matrices that commutes

with the matrix $\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$.

Follow the same procedure I did in setting up the problem and write up a presentation in paragraph form using complete sentences. You will end up with four equations similar to the ones in the example. Explain where these equations come from and why they lead to an answer that describes all matrices that commute with the given matrix.

If you choose to type your homework, you can use the Microsoft equation editor to make the matrices. If you choose to write your homework by hand please make sure that it is NEAT and LEGIBLE.